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Economic demography in fuzzy spatial dilemmas and power laws

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Abstract. Adaptive agents, playing the iterated Prisoner's Dilemma (IPD) in a two-dimensional spatial setting and governed by Pavlovian strategies ("higher success-higher chance to stay"), are used to approach the problem of cooperation between self-interested individuals from a novel angle: We investigate the effect of different possible measures of success (MS) used by players to asses their performance in the game. These MS involve quantities such as: the player's utilities U, his cumulative score (or "capital") W, his neighborhood "welfare", etc. To handle an imprecise concept like "success" the agents use fuzzy logic. The degree of cooperation, the "economic demography" and the "efficiency" attained by the system depend dramatically on the MS. Specifically, patterns of "segregation" or "exploitation" are observed for some MS. On the other hand, power laws, that may be interpreted as signatures of critical self-organization (SOC), constitute a common feature for all the MS.

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Cooperation among individuals is necessary in order to allow organizational structures that offer important advantages to them. The problem is that in general, by definition, individuals are self-interested. However, there are many examples in the animal world of cooperative behavior. Furthermore, cooperation seems to be crucial to explain several landmarks in the evolution of live organisms, from prebiotic chemistry through to the origins of human societies [1].

A particular useful conceptual playground is the *iterated Prisoner's Dilemma* (IPD) game. The PD involves 2 players, each confronting 2 choices, to cooperate (C) or to defect (D). A 2×2 matrix specifies the 4 possible payoffs for each player: A player who plays C gets the "reward" R or the "sucker's payoff" S depending if his opponent plays C or D respectively, while if he plays D he gets the "temptation to defect" T or the "punishment" P depending if his opponent plays C or D respectively. These four payoffs obey the relations: $T > R > P > S$ and $2R > S + T$. The dilemma is that in any one round, independently of what the other player does, D yields a higher payoff than C.

In general, the problem of how cooperation emerges and becomes stable is approached from a Darwinian evolutionary perspective. A central concept is the so-called *evolutionary stable strategy* (ESS) [2,3]: a strategy which if adopted by all members of a population cannot be invaded by a mutant strategy through natural selection. The evolutionary game theory spread to economics and social sciences since the early eighties computer tournaments organized by Axelrod [4]. Different mechanisms have been proposed to explain the evolution of cooperation. Perhaps the most popular is direct reciprocity, which requires either memory of previous interactions [4] or "tags" [5] permitting cooperators and defectors to distinguish one another. An alternative mechanism proposed in reference [6] has shown that spatial effects by themselves, in a classic Darwinian setting, can be sufficient for the evolution of cooperation.

Here, we consider a system of adaptive agents playing the IPD, in a two dimensional spatial setting, governed by a generalization of the strategy of "win-stay, lose-shift", known as Pavlov [7], and we explore the effect of using different *measures of success* (MS) i.e. criteria to assess the individual performance in the game. Pavlov's strategy seems to be a widespread strategy in nature [8]. In particular, experiments with humans have shown that a great fraction of individuals indeed use Pavlovian strategies [9]. Last but not least, Pavlov also has shown its efficiency when competing with several other strategies [10]. Therefore, we take Pavlov for granted and we focus on the richness of different (dynamic) equilibrium or steady states produced by distinct plausible MS. An approach similar in spirit, but without spatial structure (players were chosen

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at random) and only considering the simplest Pavlovian strategy, was proposed recently [11,12].

One of the main criticisms to agent-model approaches is that these binary and completely deterministic agents clearly are an over-simplification of real individuals, whose levels of cooperation exhibit a continuous gamma of values. We take into account the stochastic component in the adaptive behavior of real players through the use of fuzzy logic [13,14] which provides a meaningful representation of imprecise or vague concepts like "success" in an exact mathematical manner. In our work the fuzziness enters through the MS used by the players to assess their degree of success through a *membership function* μ_{success} which takes values in the interval [0,1] (0 means complete failure and 1 complete success). Each player updates his behavior (C or D) as follows: he maintains it with probability equal to μ_{success}.

We want to remark that, although we will employ the economic parlance and refer to "utilities" and "capital" as synonyms of the score the players get in each game and their cumul-ative score respectively, our agents do not necessarily represent the "homo economicus". Indeed, the agents don't need to be intelligent, as the exciting discovery that virus may also engage in simple two-player games [15] points out. In biological contexts, the score might represent the "fitness".

An arbitrary agent, represented by a square cell with center at (x, y), will play against some other agent belonging to his neighborhood $N(x, y)$. We associate two variables to it at time t: a behavioral variable $c(x, y; t)$, equal to 1 for C-agents or equal to 0 for D-agents, and his cumulative "capital" or "wealth" $W(x, y; t)$ (the sum of utilities collected up to time t). $c(x, y; t)$ is updated according to a Pavlovian strategy using 4 different MS (later we will present an example of a more sophisticated two-level-of-decision strategy that combines different MS):

- *Individual utilities IU* (ordinary Pavlov). Each player takes into account just his utilities U (R, S, T or P) in the last round. We choose $\mu_{IU}(T) = 1$, $\mu_{IU}(R) = 2/3$, $\mu_{IU}(P) = 1/3$ and $\mu_{IU}(S) = 0$.
- *Individual capital IC.* Each player compares his capital $W(x, y; t)$ with an average capital $\langle W \rangle$. Two possibilities are considered:
  - IC_L: $\langle W \rangle \equiv W_N^{av}$ i.e. the average is performed over the set of cells $(x, y) \cup N(x, y)$
  - IC_G: $\langle W \rangle \equiv W^{av}$ i.e. $\langle W \rangle$ is the global mean capital – averaged over all cells.
- *Neighbourhood welfare NW*: compares W_N^{av} with W^{av}.

For the last 3 MS, if W_{max} and W_{min} are the maximum and minimum capital among the N_{ag} agents [16], the membership function is thus chosen as:

$$\mu(X) = 1 \qquad \text{if} \quad (W_{max} + \langle W \rangle)/2 < X$$

$$\mu(X) = 2/3 \qquad \text{if} \quad \langle W \rangle < X \leq (W_{max} + \langle W \rangle)/2$$

$$\mu(X) = 1/3 \qquad \text{if} \quad (W_{min} + \langle W \rangle)/2 < X \leq \langle W \rangle$$

$$\mu(X) = 0 \qquad \text{if} \quad X \leq (W_{min} + \langle W \rangle)/2$$

where $X \equiv W(x, y)$ ($X \equiv W_N$) for measures of success IC_L and IC_G (for meassure of success NW) and $\langle W \rangle \equiv W_N^{av}$ ($\langle W \rangle \equiv W^{av}$) for measure of success IC_L (IC_G and NW).

It turns out that different MS lead to different equilibrium fractions of C-agents c_{eq}, different spatial distributions of c and W and different "economic efficiencies".

All simulations were carried out according the following a standard procedure. All the players adopt the same strategy and same measure of success. The initial state at $t = 0$ is taken as C and D chosen at random for each cell i.e. the initial fraction of cooperators c is equal to 0.5. The population varied form $N_{ag} = 2,500$ (50×50 lattice) to 1 000 000 (1000×1000 lattice). We use periodic boundary conditions. We consider evolution over $N_s = 400$ lattice sweeps ($\tau = 1\,000\,000$ to $400\,000\,000$). The grid is swept sequentially starting at $t = 0$ by the agent located at cell ($x = 1$, $y = 1$) and at each time step the agent at (x, y) plays the PD only with one of his nearest neighbors chosen at random. We considered the von Neumann neighborhood ($z = 4$ neighbor cells, the cell above and below, right and left from a given cell). On average each agent plays two times per lattice sweep (one for sure plus another one with each of his z neighbors with probability $1/z$). Therefore the average capital accumulated during N_s lattice sweeps is given by $U \times 2 \times N_s$, where U denotes the average utilities per round. The payoff matrix we consider is:

$$M = \begin{bmatrix} (1,1) & (-2,2) \\ (2,-2) & (-1,1) \end{bmatrix}$$

i.e. $R = 1 = -P$ and $T = 2 = -S$. The state of players is updated after every pairwise game, i.e. we have an asynchronous cellular automata [17].

We compute different quantities that provide relevant information about the states in which the system self-organizes. The simplest ones are: the fraction of C-agents $c(t)$ and the average capital $W^{av}(t)$. Spatial fluctuations of wealth are measured through the 2-point correlation function $G_W^{(2)}(r) \equiv \langle W(n)\, W(n+r) \rangle - \langle W(n) \rangle \langle W(n+r) \rangle$ (averages are over all positions $n \equiv (x, y)$). The size distribution of clusters of C-agents (D-agents) $N_C(s)$ ($N_D(s)$), and of agents with W over W^{av}, $N_W(s)$ are also measured. In order to detect temporal correlation, during transients, the time behavioral self-correlation $G_c(t)$ was measured and the power spectrum $P(f)$ (the absolute value of its Fourier transform) computed.

Once equilibrium has been reached, the transitions from D to C, on average, must equal those from C to D. In the case of measure IU this leads to a simple algebraic

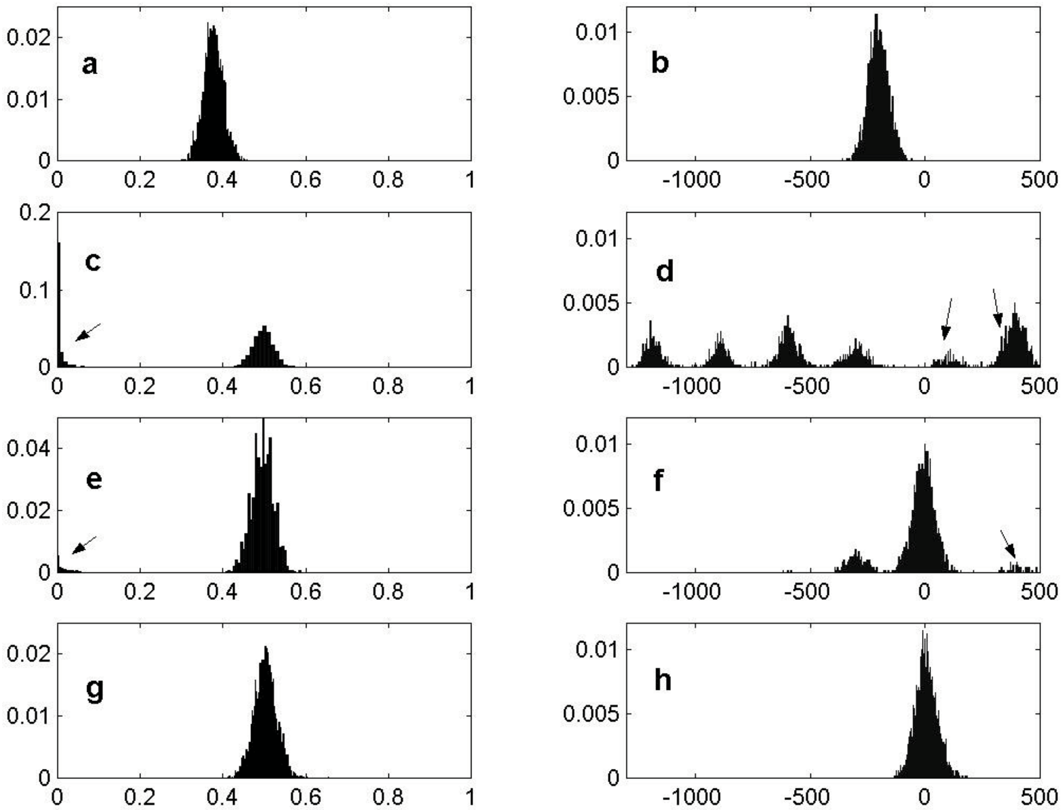

Fig. 1. Histograms for temporal averaged c (right column) and average W (left column). (a) & (b) IU, (c) & (d) IC_L, (e) & (f) IC_G and (g) & (h) NW. The c and W histograms for measures IC_L and IC_G are multi-peaked. The c histograms exhibit two peaks, one of them at $c = 0$ – large (very small) for IC_L (IC_G) (see arrows in Figs. 2c and 2e) –, corresponding to D-agents, and the other centered around $c = 0.5$. The peaks of the respective W histograms can be explained in terms of different local spatial patterns for c (see text).

Table 1. Summary of equilibrium properties for the 4 measures of success.

MS	c_{eq}	U_{eq}	W histogram	c histogram	ξ_W	W spatial Patterns	$N_C(s)$	$N_D(s)$	$N_W(s)$	$P(f)$
IU	≅0.37	≅−0.28	Gaussian	Gaussian	≅0.5	NO (random)	$s^{-1.68\pm0.03}$	$s^{-1.59\pm0.02}$	$s^{-1.52\pm0.02}$	$f^{-1.78\pm0.03}$
IC_L	≅0.3	≅−0.4	6 peaks	2 peaks	≅2	"Chess board" patches	NO power Law	NO power law	NO power Law	$f^{-1.79\pm0.01}$
IC_G	=0.5−	$=0^-$	3 peaks	2 peaks	≅1	"flowers"	$s^{-1.86\pm0.04}$	$s^{-1.60\pm0.02}$	$s^{-1.67\pm0.03}$	$f^{-1.72\pm0.05}$
NW	=0.5+	$=0^+$	Gaussian	Gaussian	≅0.5	NO (random)	$s^{-1.67\pm0.03}$	$s^{-1.62\pm0.03}$	$S^{-1.53\pm0.05}$	$f^{-1.5\pm0.1}$

Note: ξ_W is the correlation length in lattice units obtained from $G_W^2(r) \equiv \langle W(n)\, W(n+r)\rangle - \langle W(s)\rangle \langle W(n+r)\rangle$.

equation from which c_{eq} can be computed exactly, giving $c_{eq} \cong 0.36$ which agrees quite well with the asymptotic value of 0.37 found in simulations and reported in Table 1. The knowledge of c_{eq} allows to estimate the corresponding average equilibrium per-capita-utilities as $U_{eq} = 2\, c_{eq} - 1$. Hence, U_{eq} is greater (smaller) than zero if c_{eq} is greater (smaller) than 1/2.

Note the striking differences in the fraction of cooperators and capital distributions between the two IC measures in Figure 1. The "innocent" change of replacing the local average capital for the global one as a reference point to compare the individual wealth has dramatic consequences. Comparison with a global average – which comprises much information – produces a more fair (with

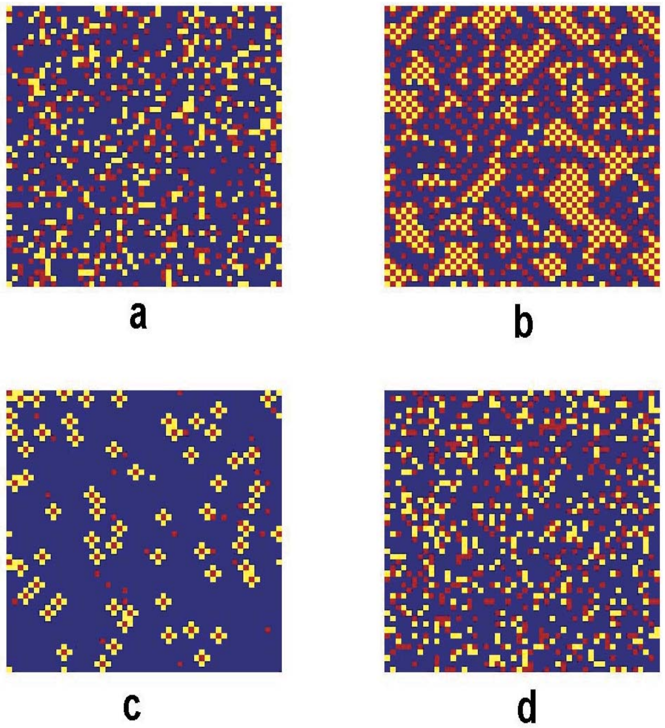

Fig. 2. Asymptotic capital maps for different MS (50×50 subsets of 500×500 lattices): (a) *IU*, (b) IC_L, (c) IC_G and (d) *NW*. We observe spatial patterns for *IC* measures, which produce non Gaussian (multi-peaked) histograms, whilst measures *IU* and *NW* produce random spatial structure.

roughly half of the population above the $W = 0$ "poverty line") and more efficient society (higher W^{av}).

In spite the similarity in efficiency between measures IC_G and *NW*, the capital distributions are very different. Something similar happens for the pair *IU* and IC_L.

We found useful a rough classification of agents into 3 "economic classes": "rich" ("poor") agents are those whose capital is greater (smaller) than $W^{av} + \sigma_W$ ($W^{av} - \sigma_W$), where σ_W is the standard deviation for W, the rest constitute the "medium class". Figure 2 illustrates the corresponding equilibrium 3-colour "maps" for W (similar maps can be generated for c). The *IC* measures exhibit complex long lived spatial patterns, reminiscent of Wolfram's class IV cellular automata [18] while *IU* and *NW* don't. The measure IC_L gives rise to "patches" of "chess board" of rich (red) and poor agents (yellow) separated by medium class agents (blue). On the other hand, the measure IC_G gives rise to "flowers", with a rich agent (red) in the center surrounded by 4 poor agents (yellow), in a sea of medium class (blue).

These spatial patterns are consistent with the respective W histograms. Specifically, the peaks of the W histograms can be explained in terms of the different stable neighborhood configurations for the behavioral variable c. For instance, the right peak at $W = 400$ (large in Fig. 1d and small signaled by an arrow in Fig. 1f) corresponds to a D player surrounded by four players with average c equal to $1/2$. This configuration gives to the central player average utilities $U = (T + P)/2 = 0.5$ per game. On average each agent plays two times per lattice sweep (one for sure plus another one with each of his z neighbors with probability $1/z$). Therefore the average capital accumulated during N_s lattice sweeps is given by $U \times 2 \times N_s = 400$, in accordance with Figure 1. In an analogous way all the other peaks can be explained. The stability of the local spatial patterns for c depend on the MS. That explain why the number of peaks for IC_L and IC_G are different in Figure 1 (6 vs. 3 peaks).

Figure 3 shows the cluster size distribution $N_C(s)$, $N_D(s)$ and $N_W(s)$ for the 4 different MS measured for 1000×1000 lattices. Except for MS IC_L, these 3 distributions exhibit power-law scaling, implying thus scale free phenomena. The absence of power law scaling for IC_L is connected with the underlying chessboard structure; for instance, from Figure 3b we can see that this MS produces only very small clusters of C-agents. It is striking that the exponent for the 3 distributions is roughly the same (see Tab. 1) [19].

We also found that the scaling of $P(f)$ is consistent with power law behavior for all MS, this, may be interpreted as an additional signature of *Self Organized Criticality* (SOC).

In addition to combinations of Pavlov plus a simple MS we studied a two-level of decision strategy with a different MS for each level: The first level consists in assessing preliminarily the performance according to IC_L. In a subsequent level, if the player did badly he updates his state using *IU*. Otherwise he behaves more nicely and looks for the combined utilities of both players. Figure 4 shows the corresponding equilibrium capital map. Note that this discriminating strategy gives rise to a segregation pattern with clusters of rich C-agents (red) and poor D-agents (yellow) in a sea of medium class (blue).

Summarizing, the combination of spatial structure and different MS produces a great diversity of demographic and spatial patterns. In particular we found: 1) consistent signatures of SOC, at least for 3 of the 4 elementary considered MS while for the IC_L measure mixed results were found, and 2) that the MS which take into account the individual wealth lead to well defined economic classes (separated peaks in W histograms). The corresponding spatial patterns show economic "exploitation". The more sophisticated two-level strategy tends to lump together the rich C-agents and, separated from them, poor D-agents leading to a higher degree of cooperation and higher efficiency (higher U_{eq}). This "segregation" pattern seems connected with a main issue in Social Sciences like the formation of *social capital*, understood as the networks and norms of trust and reciprocity that promote civic cooperation [20]. In particular, it has been argued that trust and social capital are dominant determinants of firm size across countries [21]. Our model might serve to test this hypothesis. This approach, besides its obvious applications in economics and social sciences, might be useful in other fields like ecology. The design of devices or networks, involving many interacting units, that conform to desired specifications is another possible application.

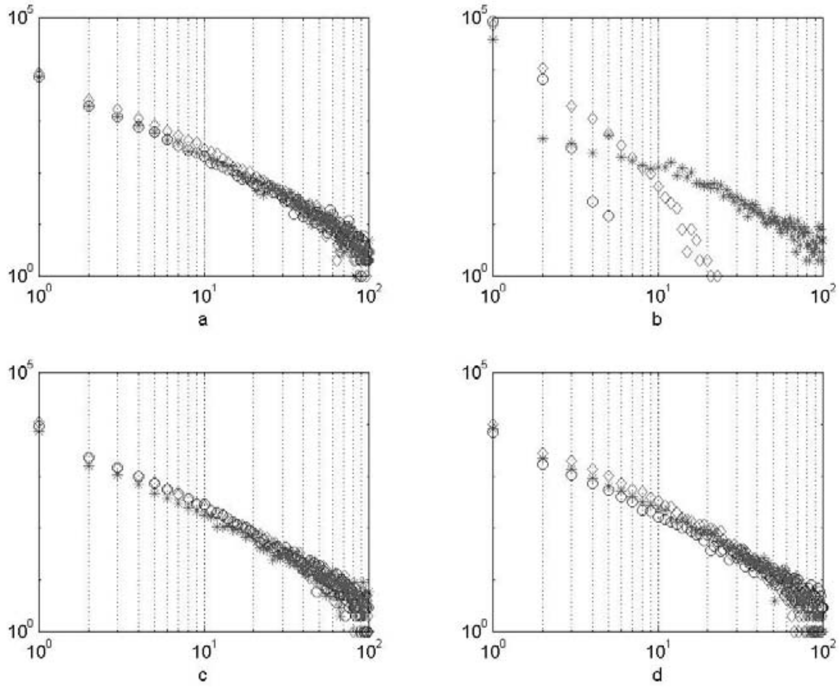

Fig. 3. Number of clusters of C-agents (o), D-agents ($*$) and agents with $W > W^{av}$ ($\diamondsuit$) vs. size of the clusters for different MS: (a) *IU*, (b) IC_L, (c) IC_G and (d) *NW*.

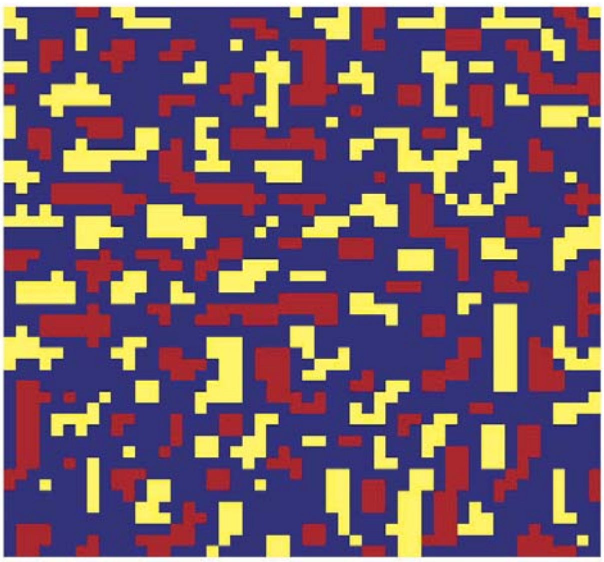

Fig. 4. Asymptotic capital map for the two level strategy. A segregation pattern with "islands" of rich C-agents (red) and "islands" of poor D-agents (yellow) in a sea of medium class (blue) is clear.

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